theorem may take the place of the longer one usually given:

"Since y varies with x, multiplying x alone will multiply y by the same factor; and similarly, multiplying z alone will multiply y; hence multiplying both x and z will twice multiply y, once by each of the respective factors. Hence x and z must enter as factors of the value of y, and since there are no other variable factors, y equals a constant expression times xz. That is $y=mxz$."

I suppose this subject of variation is pretty generally omitted by Preparatory and High School classes in Algebra. It seems to me to furnish an excellent opportunity to emphasize the difference between constant and variable quantities; a distinction the student usually meets here for the first time. I think it repays a few days careful work, by the introduction it thus gives to Analytic Geometry and the Calculus. Besides, by a few obvious applications to Astronomy and Physics, it can be made of interest to the pupil.

Such an oasis, after travelling in the desert of Radicals and Imaginaries, is very welcome.

TRUE PROPOSITIONS NOT INVALIDATED BY DEFECTIVE PROOFS.

By Professor John N. Lyle, Ph. D., Westminster College, Fulton, Missouri.

A bad cause may be brilliantly advocated and a good one poorly defended. A false proposition may be supported by plausible arguments and a true one by defective and even erroneous proofs. The true proposition is not thereby shown to be false or unworthy of acceptance.

John Playfair's demonstration of the angle-sum of a rectilineal triangle may be unsatisfactory and yet the proposition that the angle-sum is two right angles may be rigorously true and its contradictory absolutely false.

Legendre's demonstration that the angle-sum can not be less than two right angles is said by Professor Halsted to be "disgraceful." Even if this be admitted, it does not follow that the proposition itself should be doubted or rejected.

Discrediting Legendre's demonstration furnishes no legitimate warrant for postulating the truth of the hypothesis that the angle-sum can be less than two right angles.

The proofs that the angle-sum can be neither greater nor less than two right angles given in the pamphlet—Euclid and the Anti-Euclidians—may fall below the standard required by rigid geometrical science, but this does not justify the acceptance as true of the assumption that the angle-sum is greater or less than two right angles.

Lobatschewsky's theorem that the angle sum can not be greater than

two right angles is manifestly in conflict with the doctrine of those metageometers who maintain that *the space in which we dwell* has constant, positive curvature and that the angle-sum of the rectilineal triangle drawn therein is greater than two right angles.

If it is maintained that the conclusions of Lobatschewsky, Riemann and Euclid are consistent with their respective premises, the question arises which of these systems is true. If any one does not really know which is right, confession of one's ignorance may be good for the soul, but can hardly be received as satisfactory evidence that the agnostic is in possession of geometrical science.

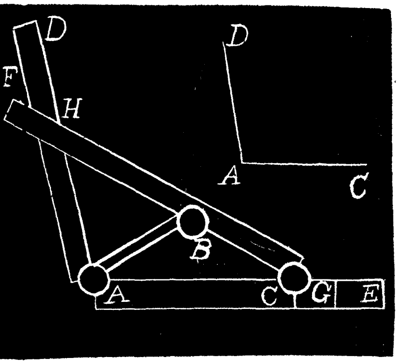
The hypothesis that Lobatschewsky, Euclid and Riemann all three tell the truth is confronted with the difficulty that they contradict each other.

Professor Halsted teaches as sound geometry the views of each of these three writers. I can not accept this teaching. If the Euclidian doctrine is true, according to logical law that which contradicts it must be false. This procedure of Professor Halsted antagonized the logical laws of non-contradiction and excluded Middle whether he is aware of it or not.

A TRISECTOR OF ANGLES.

By M. A. GRUBER, A. M., War Department, Washington, D. C.

Description. A, B, and C are centers and joints. G is a slide moving along the rule AE. The joint C is fixed to the slide so that the center C moves in the line AC. FC is a rule finely and accurately graduated from B to F, and fixed to the slide G by the joint C. AD is a fine and accurately graduated rule fixed to the rule AE by the joint A AB is a small rule jointed at A and B.



Line AB equals line BC, both remaining constant.

The *edges* of the rules for use are those radiating from the centers.

Use. It is desired to trisect the $\angle DAC$.

Place the center A of the trisector upon the vertex A of the angle, so that the edge AC of the rule AE coincides with the side AC of the angle. Then move the rule AD until the edge coincides with the side AD of the angle. Now move the slide G until BH on the rule FC equals AH on the rule AD. Then draw a line along edge of rule AB.

$\angle BAC = \frac{1}{3} \angle DAC$. Bisect $\angle DAB$ and the trisection is complete.